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Dynamic attribution of water quality indexes in a multi-reservoir optimization model

Giovanni M. Sechi, Andrea Sulis*

Hydraulic Sector, Department of Land Engineering, University of Cagliari, Italy
Tel. +39 (070) 675-5303; Fax +39 (070) 675-5310; email: andrea.sulis@unica.it

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Abstract

In the southern regions of Mediterranean Europe, the greatest part of water resources for supply systems come from artificial reservoirs. Eutrophication is one of the most serious problems affecting the quality of water stored in reservoirs. A simplified approach that includes water quality aspects as water use limiting factors in a multi-reservoir optimization model can be achieved by adoption of the Trophic State Index (TSI). This paper makes some improvements on the optimization model already presented by Sechi and Sulis in the management of large systems. Particularly, it addresses the possibility of dynamic attribution of quality indexes in the LP model. The application of the optimization approach to different operating rules in a real multi-reservoir system in southern Sardinia highlights the need for joint consideration of quality and quantity aspects for effective water management.

Keywords: Complex water systems; Trophic status; Optimization model

1. Introduction

Eutrophication is one of the most serious problems affecting the quality of water in multi-reservoir systems. The increase in nutrients leads to greater productivity of the water system which may lead to excessive increase in algal biomass or other primary producers such as macrophytes. Excessive algal biomass can seriously affect water quality, especially if it creates anaerobic conditions. Therefore, even when using a simplified approach in a mathematical optimization tool, there is a requirement to include water quality indexes associated with the trophic state of reservoirs [1].

*Corresponding author.
This paper can be considered as a development of the optimization approach presented by Sechi and Sulis [2]. It is a modelling tool intended to assist decision-makers in identifying and evaluating management alternatives when considering simplified forms of water quality classification.

It is well known that mathematical optimization procedures for large water resource systems are still unable to deal with all real-world complexities even when they can be easily incorporated in a simulation model. Nevertheless, optimization results can be seen as a reference target for simulation since optimization results can be considered as obtained by an ideal system manager [3]. Many studies have been carried out on the development of optimization models for multiple reservoir systems. However, few of them have taken water quality as an objective. Dandy and Crawley [4] modified an existing linear programming model to identify policies minimizing average salinity in water supply. A network optimization model for water allocation to demands with different quality requirements was described by Mehrez et al. [5]. Hayes et al. [6] integrated water quantity and quality modelling in an operational model for use in multi-reservoir hydropower systems.

An essential step in the construction of an advanced optimization model for addressing real problems is linked to the definition of water quality in water bodies and the identification of constraints so as to set out correct system management criteria. In Sechi and Sulis [2], a criterion for classifying reservoirs in multiple reservoir systems was defined using Carlson’s Trophic State Index [1]. Trophic State Index (TSI), which in recent years appears to have attained general acceptance by the limnological community, can be evaluated using chlorophyll-a, total phosphorus and Secchi disk transparency measurements. In Sechi and Sulis [2] the formulation of water quality constraints was addressed using chlorophyll-a measurements by means of which it is possible to evaluate the TSI(Chl) trophic index. In this paper an improvement on that approach is presented. It considers the dynamic introduction of quality index values in the optimization model. Instead of static introduction of quality values, the proposed model defines TSI(Chl) values according to changes in stored volumes in reservoirs. Two different operating rules are tested to highlight significant increases in system performance when water quality is considered as well as quantity in a common system management strategy.

2. Split-storage graph for quality index attribution

The paper describes a linear optimization tool for multiple reservoir systems under water scarcity conditions and different water uses (urban, irrigation, industrial, and hydroelectric). This tool was developed using modeling methods appropriate for the structural complexity of the optimization problem.

In the optimization approach presented by Sechi and Sulis [2] the physical system status in a single period is represented by a direct network (basic graph) consisting of nodes and arcs. A dynamic multi-period network is generated by replicating the basic graph for each period \( t = 1, T \) in the time-horizon \( T \) and then connecting the corresponding reservoir nodes for different consecutive periods. Objective function (OF) in the model incorporates operation, maintenance and repair (OMR) costs, user-defined costs to assure efficiency of the system and purification costs. Purification costs can be either fixed or dependent on water quality. As usual, optimization model constraints are introduced in order to take into account relationships between system variables: continuity equations at the nodes, relationships between the flow variables and project variables, bounds attributed to these two sets of variables and, finally, water quality constraints. In particular, for each reservoir, quality constraints
require that reservoir sources, seen overall, guarantee higher quality than (or at least the same quality as) standard quality in downstream uses. No stricter water quality constraint has been assumed, since the model considers only non-diffusive and conservative water quality processes. In Sechi and Sulis [2] the major shortcoming of this constraint is its linearity. Linearity implies that flow values can be a variable whereas quality indexes must be constrained by pre-assigned values. The quality indexes are totally independent of stored volume in reservoirs even when the proposed linear model is implemented in a scenario analysis environment [7]. Scenario analysis addresses uncertainty in water quality by taking into account a set of different scenarios corresponding to possible different evolutions of quality indexes over time.

In order to overcome this shortcoming, the LP model can be modified to consider the dependency of TSI(Chl) values on storage volume. In previous papers [2,8] TSI(Chl) and storage volume were considered as stochastically independent. However, intensive monitoring programs implemented in different countries indicate different statistical behavior of TSI(Chl) values with different total storage values. Evaluations of TSI(Chl) in these countries confirm that mean, minimum, and maximum TSI(Chl) values depend on whether stored water is above or below threshold storage values. On the basis of TSI(Chl), threshold value can be evaluated using graphical displays subjectively in a trial and error procedure or considering the joint probability distribution of storage and TSI variable. The threshold storage value is significant for TSI(Chl) characterization even when the water bodies are not deeply stratified. Several threshold values can characterize water quality in each reservoir.

Considering reservoir \( j \) at time period \( t = (1,T) \), threshold storage values are set at \( C^{(k)}_j, k = (1,K) \). The current storage \( V_j \) defines different possible states for reservoir \( j \) at period \( t \), above or below threshold value \( C^{(k)}_j \). Each storage zone limited by \( C^{(k-1)}_j \) and \( C^{(k)}_j \) is characterized by statistical values of TSI(Chl)\(^{(k)}\) (minimum, mean, and maximum values). Using this approach each reservoir \( j \) at time period \( t \) is represented by a system of \( K \) parallel reservoirs, each reservoir \( k \) representing the quality state condition of reservoir \( j \) under different storage conditions. For instance, if at time period \( t \) the storage volume in reservoir \( j \) involves the storage zone \( k \), then the water quality of the reservoir \( j \) is represented by the statistics associated with reservoir-node \( k \). To model this approach, the basic graph [2] was modified using the split-storage graph shown in Fig. 1 for three time steps and four storage zones.

When the model is applied to a single reservoir, in a single time period \( t \) the objective function (OF) becomes [2]:

\[
\min \gamma Y + \left( \sum_{k=1}^{4} c_{s,k} V_k + c_{i,k} x_{i,k} \right) \tag{1}
\]

where \( Y \) is a set of project variables with associated costs equal to \( \gamma \); \( V_k \) is the storage volume in reservoir node \( k \) (\( V_k < C^{(k)} - C^{(k-1)} \)); \( c_{s,k} \) is the unit cost of stored volume; \( x_{i,k} \) is the flow variable along the arc \( i \) away from the reservoir-node \( k \); and \( c_{i,k} \) is the cost associated to flow \( x_{i,k} \).

Flow cost configuration in OF is a key aspect in the split-storage graph model. In order to model TSI(Chl) evolution correctly, \( c_{s,k} \) and \( c_{i,k} \) should satisfy the following ranking criteria:

\[
c_{s,1} << c_{s,2} << c_{s,3} << c_{s,4}
\]

\[
c_{i,1} >> c_{i,2} >> c_{i,3} >> c_{i,4} \tag{2}
\]

This approach does not strictly aim to represent the stratification behavior of reservoirs in terms of water quality. When the model is calibrated and validated for a large storage condition, it can be used in a simple linear optimization approach, to better characterize the TSI(Chl) attribution to the reservoir in different future hydrological scenarios.
In the application to real systems, storage zones can be defined on the basis of historical TSI(Chl) and storage values in the reservoir at different seasonal steps. Using threshold values, the proposed linear optimization model with split-storage graph can reproduce the TSI(Chl) evolution in the reservoir even for generated hydrological time series.

3. Case study

Application of the optimization model considering the reservoir split-storage graph for quality modeling was carried out for the Flumendosa-Campidano water system. The system extends over southeastern Sardinia (Italy), reaching the centre of the island. Hydrology is typically Mediterranean, with alternation of severe droughts with intense rainfall periods. A detailed description of the main characteristics of the system is available in reports of European Union Project SEDEMED [9]. The system is composed of:

- ten reservoirs with a total capacity of $723 \times 10^6$ m$^3$;
- ten civil demand centers with a total request of $116 \times 10^6$ m$^3$/year;
- nine irrigation demands with a total request of $224 \times 10^6$ m$^3$/year;
- five pumping stations with a total capacity of $441 \times 10^6$ m$^3$/year;
- two industrial demands with a total request of $19 \times 10^6$ m$^3$/year;
- nine water treatment plants with total treatment capacity of $116 \times 10^6$ m$^3$/year;
- one wastewater treatment plant with total treatment capacity of $23 \times 10^6$ m$^3$/year.

Monthly data on river inflows to reservoirs are available for the period 1922–1992 (Table 1). Water demand scenarios are also available from the system manager’s (Ente Autonomo del Flumendosa, EAF) planning studies. Measurements of the main chemical and biological parameters in the reservoirs of the system were also made by EAF. Sampling for chlorophyll-a were made at least once a month from January 1994 to December 2003 at different depths in the reservoirs. On the basis of TSI(Chl) evaluated for the Flumendosa-Campidano reservoirs, a single storage threshold value ($C_{j}^{4-1}$) was evaluated at each reservoir to characterize water quality. It defines two different possible reservoir states at time $t$, as current storage at time $t$ can be above or below the threshold value.
Table 1
Flumendosa-Campidano inflow to reservoir and diversion sections

<table>
<thead>
<tr>
<th></th>
<th>Min (10^6 m³/y)</th>
<th>Max (10^6 m³/yr)</th>
<th>Mean (10^6 m³/yr)</th>
<th>SD (10^6 m³/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical series (71 y)</td>
<td>134.80</td>
<td>1032.69</td>
<td>438.51</td>
<td>239.35</td>
</tr>
</tbody>
</table>

Table 2
Threshold storage and maximum capacity in the main reservoirs

<table>
<thead>
<tr>
<th></th>
<th>Flumendosa</th>
<th>Mulargia</th>
<th>Is Barroccus</th>
<th>Cixerri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold storage [10^6 m³]</td>
<td>100</td>
<td>150</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Maximum capacity [10^6 m³]</td>
<td>316</td>
<td>347</td>
<td>14</td>
<td>32</td>
</tr>
</tbody>
</table>

Fig. 2. Relationship between storage volume and TSI(Chl) in the Is Barroccus reservoir.

For example, threshold storage equal to 5 Mm³ for the Is Barroccus reservoir was subjectively assigned based on measured values reported in Fig. 2. More complex statistical approaches can also be used. Threshold values assigned to the four main reservoirs of the system are summarized in Table 2.

To address water system optimization in the framework of different synthetic hydrologic and demand scenarios, this paper illustrates a modified version of the approach proposed in Sechi and Sulis [2] for quality index attribution to reservoirs. Hydrologic and demand scenarios 54 years length for the Flumendosa-Campidano system have been reconstructed using the official hydrologic database [10].

Attribution of TSI(Chl) values to the reservoirs in the absence of any kind of direct observation is a difficult task and a high level of uncertainty is to be expected when providing synthetically reconstructed quality data. Several types of relations based on regression analysis were considered with the aim of modeling the dependency of TSI on other data characterizing reservoirs. Here a simple least-squares curve was determined between hydrological parameters and TSI values at each reservoir. More complex approaches do not produce significantly better results, at least for the reservoirs under consideration.

The equations used to model the relationship between TSI and hydrological input to reservoirs are multiple linear regressions with monthly variability:

\[ TSI_t^{(1)} = a_1 \cdot TSI_{(i-1)} + b_1 \cdot T + c_1 \cdot HI_t + d_1 \cdot \sum_{i=1}^{N} HI_{t-i}^{1/2} + h_1, \quad \text{if} \quad V \leq C_{crit} \]

(3)

\[ TSI_t^{(2)} = a_2 \cdot TSI_{(i-1)} + b_2 \cdot T + c_2 \cdot HI_t + h_2 \quad \text{if} \quad V > C_{crit} \]
Table 3
Regression results for the Flumendosa, Mulargia, Is Barroccus and Cixerri reservoirs [\(V \leq C_{\text{crit}}\) (a); \(V > C_{\text{crit}}\) (b)]

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Variable</th>
<th>(R^2)</th>
<th>(\sigma)</th>
<th>(a_1)</th>
<th>(b_1)</th>
<th>(c_1)</th>
<th>(d_1)</th>
<th>(h_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flumendosa</td>
<td>TSI(_{\text{mean}})</td>
<td>0.84</td>
<td>0.51</td>
<td>0.01</td>
<td>0.12</td>
<td>1.57</td>
<td>0.08</td>
<td>-4.94</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.73</td>
<td>8.08</td>
<td>1.43</td>
<td>15.25</td>
<td>2.58</td>
<td>-0.13</td>
<td>-44.85</td>
</tr>
<tr>
<td>Mulargia</td>
<td>TSI(_{\text{mean}})</td>
<td>0.66</td>
<td>3.39</td>
<td>0.49</td>
<td>-0.51</td>
<td>-1.03</td>
<td>-0.69</td>
<td>24.32</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.71</td>
<td>6.39</td>
<td>0.73</td>
<td>6.68</td>
<td>-1.04</td>
<td>1.63</td>
<td>3.09</td>
</tr>
<tr>
<td>Is Barroccus</td>
<td>TSI(_{\text{mean}})</td>
<td>0.47</td>
<td>3.19</td>
<td>0.33</td>
<td>-5.52</td>
<td>0.84</td>
<td>-0.35</td>
<td>36.64</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.51</td>
<td>7.89</td>
<td>0.55</td>
<td>0.08</td>
<td>2.82</td>
<td>-2.26</td>
<td>20.93</td>
</tr>
<tr>
<td>Cixerri</td>
<td>TSI(_{\text{mean}})</td>
<td>0.44</td>
<td>2.26</td>
<td>0.26</td>
<td>0.67</td>
<td>0.14</td>
<td>0.21</td>
<td>51.50</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.59</td>
<td>8.57</td>
<td>0.60</td>
<td>-0.17</td>
<td>-0.32</td>
<td>-0.38</td>
<td>24.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Variable</th>
<th>(R^2)</th>
<th>(\sigma)</th>
<th>(a_2)</th>
<th>(b_2)</th>
<th>(c_2)</th>
<th>(h_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flumendosa</td>
<td>TSI(_{\text{mean}})</td>
<td>0.51</td>
<td>4.89</td>
<td>0.26</td>
<td>-5.93</td>
<td>-1.09</td>
<td>28.71</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.64</td>
<td>4.56</td>
<td>0.32</td>
<td>-1.47</td>
<td>0.84</td>
<td>-0.41</td>
</tr>
<tr>
<td>Mulargia</td>
<td>TSI(_{\text{mean}})</td>
<td>0.65</td>
<td>5.02</td>
<td>0.57</td>
<td>-7.61</td>
<td>-0.21</td>
<td>20.61</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
<td>0.65</td>
<td>3.67</td>
<td>0.33</td>
<td>3.39</td>
<td>0.14</td>
<td>17.54</td>
</tr>
<tr>
<td>Is Barroccus</td>
<td>TSI(_{\text{mean}})</td>
<td>0.57</td>
<td>3.51</td>
<td>0.23</td>
<td>-7.52</td>
<td>0.24</td>
<td>50.51</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
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<td>6.94</td>
<td>0.57</td>
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<td>26.57</td>
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<td>-0.36</td>
<td>29.63</td>
</tr>
<tr>
<td></td>
<td>TSI(_{\text{min}})</td>
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<td>3.29</td>
<td>0.48</td>
<td>0.48</td>
<td>0.23</td>
<td>34.95</td>
</tr>
</tbody>
</table>

where TSI is the evaluated variable at current time \(t\); TSI\(_{t-1}\) the evaluated variable in the previous time period; \(T\) the transformed date; \(HI_i\) the hydrological inflow in the reservoir in previous time period \(i\); \(a, b, c, d, h\) the regression coefficients; and \(C_{\text{crit}}\) the threshold value in storage volume.

Eqs. (3) can be used to model minimum (TSI\(_{\text{min}}\)) and mean (TSI\(_{\text{mean}}\)) values. Coefficient calibration is based on sampling values (TSI, hydrological input, storage volume) taken at each reservoir. Analysis has shown that, to better relate hydrologic inflows to TSI\(_{\text{min}}\) and TSI\(_{\text{mean}}\) values, time horizon \(N\) must be strictly related to each reservoir. The best choice of time horizon probably depends on the ratio between storage capacity and mean inflow, as well as on the physical, chemical and biological processes influencing water quality status at each reservoir.

Seasonal variability was reproduced by entering a transformed date in the model. The behavior of both observed TSI values (TSI\(_{\text{min}}\) and TSI\(_{\text{mean}}\)) shows that two peaks in TSI exist during the year, separated by about 6 months. The time function used is expressed as follows:

\[
T = \sin \left[ \frac{1}{2} (t - t_0) \ast \frac{2\pi}{P} \right]
\]

where \(t\) is the observation date; \(t_0\) the time of selected origin; and \(P\) the time cycle in TSI value (1 year).

Regression coefficients of Eqs. (3) were estimated using ordinary-least-squares multiple linear regression. The best equation at each reservoir was selected on the basis of the coefficient of determination, standard error of the regression and \(F\)-value. Coefficients of determination \(R^2\), residual standard errors \(\sigma\) and parameters of each relation are listed in Table 3a and b. Coefficient of determination ranges from 0.44 to 0.84 and,
for this kind of analysis, it can be considered sufficiently accurate for performing subsequent determinations [11].

Pursuant to Italian legislation, Sechi and Sulis [2] described the procedure for applying a Quality Evaluation (QE) index in the form of a single synthetic quality index varying from 1 (excellent) to 5 (bad). On the basis of TSI\textsubscript{min} and TSI\textsubscript{mean} values we attributed a minimum and a mean QE value to the water stored. Figs. 3a and 3b show the frequency distribution (expressed as percentage of 54 years) of monthly QE\textsubscript{min} and QE\textsubscript{mean} in the five classes for the reservoirs monitored.

Standard QE values (QE\textsubscript{R}) are the lowest QE values that must be guaranteed for demand centers. Modeling the Flumendosa-Campidano system, we assigned QE\textsubscript{R} equal to 1 for urban use, 3 for industrial use and 4 for irrigation use.

4. Optimization results and conclusions

Water drawn from reservoirs can be located at different depths. This allows for better quality in reservoir releases: the so-called “selective withdrawal”. By enabling control of the quantity and quality of releases, selective withdrawal is a typical reservoir-operating rule that integrates these aspects in the management of a reservoir system. When the selective withdrawal operating rule [OP(1)] is used, water released from the reservoir is characterized by a QE value equal to QE\textsubscript{mean}. On the other hand, a non-selective withdrawal operating rule [OP(2)] does not select the best layer and only considers quantity aspects. In this case, the QE value can be considered equal to QE\textsubscript{min}. These operating rules were implemented in the model proposed by Sechi and Sulis [2] by means of two different constraints.

The split-storage optimization model described above was implemented to assess improvement in system performances when considering quality and quality aspects in a common management strategy [OP(1)], instead of focusing attention on water quantity alone [OP(2)]. Optimization analysis spanned a 54-year time horizon and two optimization phases in order to highlight the consequences of examining quantity and quality issues jointly. Time reliability (expressed as percent value of the months in which deficit is equal or less than predefined thresholds) and
volumetric reliability (expressed as deficit rate on demand value for the entire period of analysis) are used to quantify system performances.

Using the non-selective operating rule [OP(2)], Table 4 summarizes the degree of reliability of system management. Poorest volumetric reliability is obtained for irrigation (58%) since urban and industrial demands are marked by the highest deficit penalization cost in the OF. Temporal reliability values are estimated considering deficit values of 0% and 25%. As was to be expected, applying a selective withdrawal operating rule led to modification of optimal flux configuration. Operation of a reservoir under quality criteria can achieve the most restrictive downstream use, called designed use [12]. System performances in Table (5) show that OP(1) guarantees designated uses better than OP(2). Industrial demand is met during the whole time horizon. Temporal and volumetric reliability in civil demand improve significantly. Persistence of a significant deficit in irrigation demand occurred. In this case, not even a selective withdrawal rule is able to ensure better performances.

Considering both storage volumes and associated quality indexes as decision variables of the problem, the proposed split-storage graph seems a good way to overcome a major obstacle to the application of linear programming in integrated quantity-quality analysis of complex water systems. Moreover, difficulties in incorporating all the complexity of the water system into the linear model still limit the effectiveness of practical application to real-world systems. Nevertheless, this approach can be viewed as a robust preliminary screening tool for providing information on operating policies. These policies will be further evaluated with more detailed simulation models not limited by many of the optimization model assumptions.

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