Balancing cost-risk in management optimization of water resource systems under uncertainty

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A B S T R A C T

When a scarce water resource is distributed between different users by a Water Resource Management Authority (WRMA), the replenishment of this resource as well as the meeting of users’ demand is subject to considerable uncertainty. Cost optimization and risk management models can assist the WRMA in its decision about striking the balance between the level of target delivery to the users and the level of risk that this delivery will not be met. Addressing the problem as a multi-period dynamic network optimization, the proposed approach is also based on further developments in stochastic programming for scenario optimization. This approach tries to obtain a “robust” decision policy that minimizes the risk of wrong decisions when managing scarce water resources. In the paper we also illustrate two application examples for water resources management problems.

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1. Introduction

Quantitative system management models are considered for the distribution of scarce resources under uncertainty and conflicting demands of different users and activities. Specifically, we refer to models for water resources supply systems in regions where such resources are scarce; see for example Dembo (1995), Loucks et al. (1981), and Sechi et al. (2005) for discussion of the topic. However, the methods considered here are applicable to a wide range of management problems in other different areas, allowing a robust optimization model to be obtained.

One way to address this problem is to construct a set of scenarios about the future values of uncertain parameters, like the values of resource replenishment and demand processes for each future time period. Then scenario analysis is employed in order to construct the optimal decision policies for each scenario and to investigate the sensitivity of these decisions with respect to the changes in uncertain parameters. This can be a useful technique, especially when scenarios do not differ drastically from one another. In such a situation it may be possible to identify a decision policy that is relatively stable with respect to scenarios. Aspects of stochastic optimization with uncertainty described by scenario trees can be found in Dembo (1995), Rockafellar and Wets (1991), Römisch and Schultz (2001), and Pallottino et al. (2005). The stochastic optimization approach tries to obtain a “robust” decision policy that minimizes the risk of wrong decisions. It also allows exploitation of the algebraic structure of models such as those arising from dynamic networks and multi-scenario structures. In the following paragraphs, we focus on the study of multi-period systems supported by dynamic networks by integrating scenario optimization and a subsequent re-optimization phase. In the scenario optimization phase we will represent data uncertainty by a multi-scenario model, obtaining a so-called “barycentric” value of some critical variables with respect to all scenarios. Multi-scenario solutions are then included in a successive re-optimization model where the barycentric value is considered as a redefined target. This procedure gives rules when planning in relation to the risk of wrong decisions and defining emergency plans to face resource scarcity. Finally, the application to referenced reservoir-demand water system is considered to illustrate this approach.

2. Scenario optimization model for water system management

The development of the model for cost–risk balanced management of scarce resources has been done by formulating a multi-stage stochastic programming model for water system management under uncertainty. This can be done in three phases: (1) generation of scenarios for describing potential evolution of uncertain parameters and construction of a scenario tree; (2) development of a deterministic optimization model for water resources management for each scenario; (3) aggregation of these models into a multistage stochastic programming model (Birge and Louveaux, 1997; Kall and Wallace, 1994). Water resources management is a complex and relevant issue particularly when
droughts occur and the necessity of establishing preventive measures requires an effective management policy for the uncertain future of the hydrologic scenario. This problem can be modeled as a multi-period dynamic network flow problem where, in any period of the planning horizon, nodes are defined at relevant decision points, such as sources, demands, reservoirs, groundwater, diversion sites, hydropower station sites, and so on. Arcs represent activity connections and management decisions such as storing water in reservoirs and water transfer to demand centers. Source and demand forecasts are developed by estimating hydrological inflows and water transfer patterns. Management decisions have to be made sequentially on the basis of decision strategies determined with reference to a predefined scenario, where a scenario represents a possible realization of the uncertain data on the examined time horizon (Sechi and Zuddas, 2008).

Deterministic models are not adequate to describe the variability of some crucial parameters and even small differences in data in two different scenarios can produce significantly different solutions (Pallottino et al., 2005). The scenario analysis approach attempts to face the uncertainty factor by taking into account a set of different supposed scenarios corresponding to the different possible time evolutions of crucial data. Two scenarios sharing a common initial portion of data must be considered together and partially aggregated with the same decision variables for the aggregated part in order to take into account the two possible evolutions in the subsequent diverse parts. In this way, the set of parallel scenarios is aggregated by producing a tree structure called a scenario-tree. The root node of the tree corresponds to the beginning of time period \( t = 1 \). From this node, \( n_0 \) scenarios start and continue in parallel for \( T_1 \) periods. At \( t = T_1 \) each of \( n_0 \) scenarios is split into \( n_1 \) scenarios and all the obtained scenarios \( n_0 n_1 \) continue in parallel for a further \( T_2 \) periods until \( t = 2T_1 \), when each of them is split into \( n_2 \) additional scenarios.

An example of such a tree with \( T_1 = 12, T = 36, K = 3, n_0 = 1, n_1 = 3, \) and \( i = 1:2 \) is shown in Fig. 1. Usually one time period corresponds to 1 month, and we take \( T_1 = 12 \), which corresponds to 1 year. This means that splitting occurs at the end of each year, which conforms to the seasonal patterns of inflows and demands. This scenario generation and aggregation process which produces a scenario tree from individual scenarios is described in more detail in Manca et al. (2004) and Pallottino et al. (2005).

All scenarios are collected in an aggregated multistage stochastic programming model to obtain a global set of decision variables on the whole set of scenarios. Aggregation rules (Rockafellar and Wets, 1991; Pallottino et al., 2005) guarantee that the solution in any given period is independent of the information not yet available, in other words, non-anticipative constraints. A weight can be assigned to each scenario to characterize its relative importance. The weights could represent the probability of occurrence of each scenario, if this probability can be estimated by stochastic techniques or statistical tests based on historical data. The aggregated multistage stochastic programming model can be expressed as the collection of one deterministic model for each scenario \( g \in G \), plus a set of congruity constraints representing the requirement that the subsets of decision variables corresponding to the indistinguishable part in each scenario must be equal among themselves:

\[
\begin{align*}
\min & \quad \sum_{g \in G} p_g c_g x_g \\
A_g x_g &= b_g, \quad \forall g \in G \\
x_g &\leq u_g, \quad \forall g \in G \\
x &\in S
\end{align*}
\]  

where \( x_g \) represents the vector of all operation and flow variables in scenario \( g \); the vector \( c_g \) describes the unit cost of different activities like delivery cost, opportunity cost related to unsatisfied demand, opportunity cost of spilled water, and so on. The objective function is defined as the average of the cost objectives of all scenarios weighted with their probabilities \( p_g \). The set of standardized equality constraints describes the relationships between storage, usage, spill, and exchange of water at different nodes and in subsequent time periods. The right hand sides \( b_g \) are formed from scenario data of inflows and demands. The lower and upper bounds \( l_g \) and \( u_g \) are defined by structural and policy constraints on the functioning of the system. All decision variables and data are scenario dependent, hence the index \( g \). All constraints in Eq. (1) are collected from all scenarios and put in the aggregated model. An additional set of non-anticipative constraints \( x \in S \) represents the congruity constraints derived by aggregation rules. These constraints ensure that the decision variables that describe decisions at time periods when scenarios are not yet split must coincide. In other words, they ensure that decisions do not depend on the future. For example, the values of decision variables related to scenarios \( g_1 \) and \( g_2 \) from Fig. 1 must coincide during time period \([0, T_1]\) and decision variables related to scenarios \( g_2 \) and \( g_3 \) must coincide during time period \([2T_1, 3T_1]\).

In Pallottino et al. (2005) results are obtained by the solution of multi-period stochastic programming model (1) with scenario optimization and its comparison with the deterministic model in the case of one real water resources system when scarcity occurs. Making use of this approach, it is shown that scenario analysis could produce a more effective and general management policy compared to the deterministic approach. Actually, it allows mitigation of system crises such as early empty reservoirs and unfulfilled demand.

3. Balancing cost and risk in resource delivery

The cost minimization point of view in the scenario optimization model developed in the previous section may not be sufficient and should be enhanced by considering the associated risk of resource failure evaluations. For this reason, resource management models employed by the Water Resource Management Authority (WRMA) should include the balance between costs and risks to which the end users are exposed. We assume that the resource in question is scarce and for this reason the demand will not be satisfied in many scenarios. We shall denote the vector of resource delivery to the users under scenario \( g \) as \( x_g \). This will be a subvector of the vector \( x_g \) of all decisions of the WRMA under scenario \( g \). In the case when there are a number of users for which the WRMA can have different policies, we shall index these users with...
index \( l = 1 : L \). Then the vector \( x_g \) of resource delivery will be further subdivided into sub-vectors \( x_g = (x_{1g}, \ldots, x_{Lg}) \). The vector of all deliveries to the user \( l \) under all scenarios will be denoted by \( x^l = \{ x_{lg} | g \in G \} \) and the vector of all resource deliveries under all scenarios to all users will be denoted by \( \hat{x}, \hat{x} = \{ x_{lg} | g \in G \} \). In such scarcity situations, users should develop and adopt an emergency policy to alleviate and manage the effect of shortages on their activities. In order to do this a user \( l \) should know in advance the reduced target level of demand satisfaction that the WRMA is willing to deliver to him and that is independent of possible scenarios of uncertainty. Usually this new target level \( x^l \) will be less than the user’s demand due to inherent scarcity of the resource. The difference between demand and reduced target \( x^l \) will represent the planned shortage of the resource which the user \( l \) is asked to accept under drought conditions. Besides this planned shortage there can also be unplanned shortages when, due to severe lack of resources under some scenarios, the WRMA will not be able to deliver even the reduced volume \( x^l \). In order to face this situation and develop an appropriate emergency policy, the user \( l \) should be informed by the WRMA about the reliability of the delivery of the target level of resource or, in other words, the quantitative level of risk that the actual delivery will fall short of the promised one. This measure of risk for user \( l \) will be denoted by \( R(x^l, x^l) \).

The simplest way to define the target delivery is to solve the optimal management problem (1) and take the expected delivery as the target delivery:

\[
x^l = E\{\hat{x}_l\} = \sum_g p_g \hat{x}_g
\]

(2)

The shortage can be measured as the variance \( \sigma^2 = \sigma^2(x, x^l) \) of deliveries:

\[
R(x, x^l) = \sigma^2(x, x^l) = \sum_g p_g ||x_g - x^l||^2
\]

(3)

Knowing the target delivery \( x^l \) and the variance between the target and the actual delivery \( R(x, x^l) \), the system manager could set the supply policy confronting and balancing the possible resource shortages with associated costs. Nevertheless, defining the target resource delivery as the average resource delivery under cost minimization has the shortcoming that it assumes that the objective of WRMA is fully described by the cost minimization. In reality a water system is more complex because the purpose of optimal management of the distribution of a scarce resource is to achieve a balance between sustainable maintenance of the whole system and satisfaction of the demand of end users that is in some sense “reasonable.” Pure cost minimization may result in unacceptably high risk of not achieving the target delivery to the end user. For this reason the balance between the total costs and risks should be explicitly included in the optimization model. This can be done in the way in which the balance between risk and performance is modeled in financial risk management (Markowitz, 1991).

The cost–risk balancing problem can be formulated with the objective function containing both the risk and the cost terms as follows:

\[
\min_{x^l} (1 - \lambda) \sum_g p_g c_g(x_g) + \lambda \sum_g p_g d(x_g) - x^l
\]

subject to constraints (2) and (3). The parameter \( \lambda \) can vary between 0 and 1; \( \lambda = 0 \) corresponds to the pure cost minimization, while for \( \lambda = 1 \) the problem becomes one of minimization of risk. Intermediate values of \( \lambda \) provide different tradeoffs between costs and risk. In the case of Euclidean distance we have the barycentric problem:

\[
\min_{x^l} (1 - \lambda) \sum_g p_g c_g(x_g) + \lambda \sum_g p_g d(\hat{x}_g, x^l)
\]

subject to constraints (2) and (3). The parameter \( \lambda \) can vary between 0 and 1; \( \lambda = 0 \) corresponds to the pure cost minimization, while for \( \lambda = 1 \) the problem becomes one of minimization of risk. Intermediate values of \( \lambda \) provide different tradeoffs between costs and risk. In the case of Euclidean distance we have the barycentric problem:

\[
A \hat{x} - \lambda \sum_g p_g \hat{x}_g = b_g, \quad \forall g \in G
\]

(4)

\[
l_g \leq x_g \leq u_g, \quad \forall g \in G
\]

\[
x \in S
\]

Fig. 2. General tradeoff between costs and risk.

How can the appropriate value of the weight \( \lambda \) which balances the risk and costs in water systems be selected?

Since the WRMA and users are jointly responsible for the functioning of the whole system, this should be a matter of compromise between these actors. Here also we can draw some suggestions from risk management approaches developed in finance. Let us look at resource distribution decisions \( x \) as a portfolio of resource distribution. Then following the approach of portfolio theory (Markowitz, 1991) we shall construct the efficient frontier in the space of risk–cost by solving the problem (5) for different values of \( \lambda \in [0,1] \). This frontier will generally have the shape shown in Fig. 2. The cost–risk analysis for water resources systems was also previously considered by Sechi and Zuddas (1993) when suggesting an optimal expansion procedure in the system dimension to face shortage risk. This approach starts from the consideration that, having a predetermined dimension configuration of the system, an admissible flow configuration follows when minimizing the risk of deficit in the system. In the test case examined in Sechi and Zuddas (1993), the function \( g(y) \) expresses the variation in the minimum risk of shortage in the system when the configuration \( y \) of reservoirs is varied. The function \( z(y) \) expresses the cost when the configuration \( y \) is varied. Considering a simple system with a fixed demand and two reservoirs with capacity \( y_1 \) and \( y_2 \), Fig. 3 shows the representation of iso-cost (lines of equal cost) and isodeficit curves for the system. The search for an optimal design is therefore carried out along the expansion path in the direction in which the direction opposed to the gradient of \( z(y) \) agrees with the decreasing direction of \( g(y) \). This technique fundamentally applied the optimality conditions when considering economical evaluation of production alternatives (James and Lee, 1971). The non-differentiability of the \( g(y) \) isodeficit curves makes it necessary to resort to an ad hoc conjugate-direction technique to search for the optimality (Sechi and Zuddas, 1993). The cost associated with deficit risk can be found along the optimal expansion curve and the efficient frontier in the cost–risk analysis is given in Fig. 4.

In more general terms the acceptable level of risk should be negotiated between WRMA and resource users. The weight \( \lambda \) will correspond to this value of risk. The planned shortage is not included in this risk formulation, but it is included in the cost part instead. A similar cost–risk balancing methodology was also
of the planned one. The WRMA can also give the user its estimate of the worst shortage which could happen with respect to the planned delivery. This is done by the re-optimization of the planning problem (5) for the worst possible scenario $g^* \in G$ in the case when such a scenario is clearly identifiable. User demand in such a re-optimization problem will be taken to be equal to the planned delivery $x^p$; this demand will form part of the right hand side of (5). The risk term will be absent from the objective function in (5). Therefore the re-optimization problem will take the form:

$$\min_{x^g,g \in G} c_g(x^g)$$

$$A_g x^g = b_g,$$

$$l_g \leq x^g \leq u_g,$$

where $b_g$ is constructed considering the worst case and $x^p$ is in place of the original user’s demand. The solution of this problem $x^g$ will be communicated to the user as the worst that can happen with regard to the delivery of scarce resource. In the case when the worst case scenario is not clearly identifiable, the problem can be re-optimized for some subset $G^*$ of the original scenario set $G$ which is associated with the worst scenarios, again taking the planned delivery $x^p$ as the user demand. Then the worst delivery will be the smallest delivery with respect to all scenarios in $G^*$. In the limit, the re-optimization may be performed on the whole set $G$ of original scenarios with the planned delivery $x^p$ as the user’s demand and without the risk term in (5).

5. Applications

5.1. Single reservoir and demand water system scheme

To illustrate the approach, we preliminarily refer to a simple reservoir-demand scheme, drafted in Fig. 5a. The modeling approaches described in the previous sections were implemented in the prototype WARGI (Sechi et al., 2005; Sechi and Sulis, 2007) of a decision support system for the management of scarce water resources. Hydrological inflow in the reservoir and demand setting refer to a real system in the southern part of the Sardinia region, Italy. An extended description of this system is given in Sechi et al. (2005), Sechi and Sulis, 2007. We want to determine the resource management policy over a time horizon such that the known resource demand is satisfied (as far as possible) and the total deficit cost is minimized. A time-horizon of four years comprising 48 monthly time-periods from October 1970 to September 1974 is considered. The simplified single-reservoir–single-demand
The single demand center includes all the demand centers supplied by the reservoir. The total demand equals $235.2 \times 10^6$ m$^3$/year.

Configuration has been obtained by adding the capacities of the main reservoirs included in the real system up to $584.1 \times 10^6$ m$^3$. The single demand center includes all the demand centers supplied by the reservoir. The total demand equals $235.2 \times 10^6$ m$^3$/year.
To validate the approach we consider both the case of fixed demand (the same in all time-periods) and time-varying demand. Moreover, in order to provide a more transparent comparison of different solutions, we considered the case of a simplified structure of the scenario tree containing only two inflow scenarios: $g_1$ and $g_2$. As shown in Fig. 6, the branching of scenarios occurs in the 12th period. The hydrological inflows in scenario $g_2$ are obtained from the historical data. Scenario $g_2$ is assumed as a reference scenario. The scenario $g_1$ simulates a severe shortfall in water inflows. It is derived from $g_2$ in a simple way, assuming that a reduction of 50% in hydrological inflows will occur after the branching time.

The behavior of stored volumes in the reservoir and water supplied to the demand center have been obtained using a standard deterministic optimization (deterministic policy), the scenario optimization stochastic programming approach (stochastic policy), and a re-optimization phase (barycentric policy) as previously illustrated.

The barycentric target value has been obtained by solving the problem (5) for $\lambda = 0.5$. The re-optimization phase has been

Table 1
Variable demand: original (first row of figures) and barycentric redefined (second row of figures) monthly requests ($10^6$ m$^3$).

<table>
<thead>
<tr>
<th></th>
<th>October</th>
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Fig. 9. Stored water in reservoir – variable demand.

Fig. 10. Transferred resource to the demand center – variable demand.
achieved with the demand set at this barycentric target value and adopting hydrological data corresponding to scarcity scenario $g_1$.

Considering the case of a fixed demand (the same in all time-periods), Fig. 7 represents the behavior of stored volumes in the reservoir and Fig. 8 the resources delivered to the demand center. Particularly, Fig. 7 shows stored volumes in the reservoir obtained by the deterministic policy, the stochastic policy, and the barycentric policy. Deterministic policy leads to earlier emptying of the reservoir for scenario $g_1$ compared to the stochastic policy for the same scenario. The barycentric policy improves the behavior of stored volumes if the scarce scenario occurs. Decisions adopted using this approach better preserve the resource available in the reservoir in each time period, even with respect to the stochastic policy.

Supply behavior in Fig. 8 shows that stochastic policy for critical scenario $g_1$ exhibits a smoother resources distribution and a lower variation compared to the deterministic policy which will lead to an unexpected drop in supply at the end of the time horizon. Following the stochastic policy, when scenario $g_1$ occurs (blue line in the figure), the original resource demand $D$ is fulfilled until the 12th time period and the difference between $D$ and the delivered resources is the unplanned deficits $U_s$. If scenario $g_2$ occurs, the system has no shortage. Using the barycentric policy, the level of demand is set at the barycentric target value. The figure illustrates how the barycentric scenario results enhance the performance of the system.

In this barycentric re-optimization phase the reduced demand $D_b$ is set at the barycentric value $x_b = 18.47 \times 10^6$ m$^3$ per month.
and the demand is completely fulfilled until the 24th monthly period. The difference between the behavior of the delivered resource in each time period $t$ (the pink line in the figure indicates barycentric policy) and the value $D_b$ identifies the set of unplanned deficits $U_b$. Moreover, the difference between original demand $D$ and reduced demand $D_b$ can be considered as a planned deficit, identified by $P_b$ in Fig. 8.

The optimization procedures have been also developed considering demand that varies monthly, as typically occurs for irrigation requests. Table 1 reports the adopted values. Figs. 9 and 10 show the behavior of stored volumes in the reservoir and resource delivered to the demand center in the case of variable demand.

The results confirm the better performance of the stochastic approach scenario with respect to deterministic optimization in this case also. The stochastic policy scenario allows the drop in supply that occurs when using deterministic optimization to be avoided; moreover the barycentric policy allows a more regular resources distribution to be achieved in the case of variable demand. The redefined annual supply target obtained considering the barycentric policy is equal to $194.94 \times 10^6$ m$^3$; monthly values are given in Table 1.

5.2. Multiple reservoirs and demand centers scheme

The adopted multiple reservoir–demands water system is drafted in Fig. 5b. The scheme considers two reservoirs and three demands and represents in a more adherent manner the real system of South Sardinia. The scheme comprises typical in-series
reservoirs, and all demands are supplied by the downstream reservoir. Regulation capacities are $264.10 \times 10^6$ m$^3$ for reservoir S2 and $320.00 \times 10^6$ m$^3$ for reservoir S3. Civil and industrial demands are $115.7$ and $39.0 \times 10^6$ m$^3$/year respectively and are considered constant in each month. Irrigation demand is equal to $80.5 \times 10^6$ m$^3$/year and monthly distribution is proportionally the same as that reported in Table 1. Inflows in reservoirs are given in Figs. 11 and 12 when considering the historical period from October 1970 to September 1974, which is still defined as scenario $g_2$, and considering the 50% reduced scenario $g_1$. As required by the regional WRMA (RAS, 2005), in this scheme with multiple competitive demands, greater priority was given to civil and industrial uses than to irrigation. Defining the costs $c_g$ in Eqs. (4), (5), and (6), we considered a 1/10 ratio between costs related to deficits in irrigation versus costs related to civil and industrial deficits.

The higher priority given to civil and industrial uses determines that related demands are always satisfied and the deficits are assigned only to the agricultural demand using all the modeling approaches. The obtained results for reservoirs' storage behavior are reported in Figs. 13 and 14. As required by the usual system management rules, the upstream reservoir S2 tends to maintain the water resource as long as resource is still available in the downstream reservoir S3. Nevertheless scenario $g_1$ determines emptying of both reservoirs at the end of the time horizon.

Water transfers in Fig. 15 refer to the irrigation demand, as the higher priority demands are always satisfied. Estimated annual barycentric value is equal to $56.04 \times 10^6$ m$^3$/year and monthly distribution is proportionally the same as that reported in Table 1. Clearly evident from Fig. 15 are the advantages of adopting the supply procedure given by the barycentric policy compared to deterministic optimization, which could cause a complete shortage starting from period 35 if scenario $g_1$ were to occur.

6. Conclusions

An approach for considering a cost–risk balanced procedure for the management of scarce water resources in conditions of uncertainty has been proposed. This approach is based on the methodology of scenario-based stochastic programming optimization and it is developed by adapting some risk management tools from modern financial theory and investment science. In the paper the aspect of storage and supply procedure when considering the possibility of scarce hydrological inflows in reservoirs was analysed in particular. In addition to risk management, a new approach based on a re-optimization phase was developed that allows the user to arrange emergency policies by adopting the barycentric value as a new target and thus minimize the risk of drastic reduction in resources delivery.

Although the specific application is described with reference to two simple water resources systems, the presented approach can be adopted in a wide range of management problems. It appears that this approach can be very useful in order to decide on a set of planning procedures and operational measures when the system is affected by a high level of uncertainty in supply or demand pattern evolution.

References


Fig. 15. Multiple reservoirs scheme: resource transferred to the irrigation center.

